# Determination of complex thermal boundary conditions using a Particle Swarm Optimization method

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**Abstract**

The methodology based on the Particle Swarm Optimization (PSO) method, as a recent stochastic optimization technique to solve complex inverse heat transfer problems is outlined. Temporal and spatial dependent Heat Transfer coefficient obtained on the surfaces of a cylindrical work piece is recovered by solving the inverse heat conduction problem. The fitness function to be minimized by the PSO approach is defined by the deviation of the measurements and the calculated temperatures is minimized. The PSO algorithm has been parallelized and implemented on a GPU architecture. Numerical results are demonstrated that the determination of Heat Transfer Coefficient functions can be performed by using the PSO method, as well as, the GPU implementation; provide a less time consuming and accurate estimation.

**Keywords**

Inverse Heat Conduction Problem, Heat Transfer Coefficients, Particle Swarm Optimization

## 1 Introduction

Inverse heat conduction problems are known as “reverse engineering” problems, due to the reversal of a cause-effect sequence, in the field of heat transfer analysis. An inverse problem means that some of the initial, boundary conditions or material properties are not fully specified as determined from the measured temperature profiles at some specific locations. The inverse problems in most situations are likely to be ill-posed [Beck 1985]. Solutions of the inverse problem are very sensitive to measurement errors, i.e. small errors in the measured data values can produce very large errors in solutions. In general, the exclusivity and stability of an inverse problem solution is not guaranteed. In recent years, the inverse problems have been studied extensively due to their applications in various engineering disciplines.

The most of the methods approach the inverse heat conduction problem, as an optimization problem, i.e. the problem is defined as the minimization of a cost function or a fitness function measuring the distance between measurements and predictions [Alifanov 1994, Özisik 2000]. With the improvement of computer capability, a variety of numerical techniques and computational methods have been developed to provide accurate solutions for inverse heat conduction problems (IHCP) in the last decade. Among these methods, stochastic optimization methods have become a popular means of solving inverse problems, due to their capability of finding the global optimal result without computing the complicated gradient of the objective function.

Genetic algorithms [Verma 2007, Kim 2004] are applied successfully for solving many types inverse heat transfer problem. The quantitative evaluation of different numerical optimization techniques showed [Felde 2012] that stochastic methods could serve more accurate results for IHCP than gradient approaches in recovering complex thermal boundary conditions. The Particle Swarm Optimization (PSO) algorithm became popular in the recent years due to its ability of maintaining a good balance between the convergence and diversity. Applications of PSO algorithms in the field of heat transfer are still limited. An inverse application of boundary elements method to estimate the thermal conductivity and the shape of an inclusion was implemented [Ardakani 2009]. Qi et al. [Qi 2008] applied the multi-phase PSO method to solve the inverse radiation problem. The proposed method applied the benefits of both two-group PSO and multi-start PSO algorithms. The effectiveness and efficiency of Particle Swarm Optimization technique in inverse heat conduction analysis were analysed by Vakili and Gadala [Vakili 2009]. Three variations of the PSO method, i.e. basic PSO, repulsive PSO, and complete repulsive PSO, were performed to solve the boundary inverse heat conduction problem, in one, two and three dimensions. The results showed that PSO can reduce the stability problems of the classical methods, for solving the inverse heat conduction problems.

In this work, an inverse analysis for the reconstruction of local coordinate and a time-varying Heat Transfer Coefficient, in two-dimensional cylindrical coordinates is investigated. The inverse heat conduction analysis is based on the application of a PSO technique. Transient temperature measurements at multi-locations in the body of the work piece, obtained by the solution of the direct heat transfer problem, served as the virtual experimental data required to solve the inverse analysis. The fitness function which is defined by the quadratic residual between the measurements and the calculated temperatures is minimized. The PSO algorithm has been parallelized and implemented on a GPU architecture. Numerical results are demonstrated that the determination of Heat Transfer Coefficient functions can be performed by using the PSO method, as well as, the GPU implementation; provide a less time consuming and accurate estimation.

## 2 The physical and mathematical models

The determination of the Heat Transfer Coefficient is an important issue of the IHCP and has been extensively studied. Improvements and adaptations of the numerical algorithms on the applications are still an active area of research for obtaining stable and reliable results. A two-dimensional axis-symmetrical heat conduction model is considered to estimate the temperature distribution in a cylindrical work piece (the radius and length of the cylinder is noted by R and Z). The cylinder is subjected to a longitudinal local coordinate and time varying Heat Transfer Coefficient HTC(z,t) on all its surfaces. Both the thermal conductivity and the heat capacity are varying with the temperature, k(T) and $C_p$(T). The dimensional mathematical formulation of this nonlinear transient heat conduction problem can be described as follows:

$$\frac{\partial}{\partial r}\left(k\frac{\partial T}{\partial r}\right)+\frac{k}{r}\frac{\partial T}{\partial r}+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+q_v=\rho C_p\frac{\partial T}{\partial t} \tag{1}$$

with the initial and the boundary conditions

$$T(r,z,0)=T_0 \tag{2}$$

$$k\frac{\partial T}{\partial z}\bigg|_{\substack{0\le z\le Z\\ r=R}}=HTC(z,t)\left[T_q-T(r,z,t)\right] \tag{3}$$

where r and z is the local coordinate, t is the time, ρ is the density, $T_0$ is the initial temperature and $T_q$ is the temperature of the cooling medium. In this work, the weighted Schmidt explicit finite difference method is used to discretize the Eqs. (1-3) and solve the direct problem.

## 3 The inverse heat transfer problem

Assuming that the temperature inside the work piece and/or on its surface is measured during the heat transfer process, it is possible to solve the inverse heat conduction problem by determining the time/or temperature variations of the thermal boundary conditions [Beck 1985, Tikhonov 1977, Alifanov 1994, Özisik 2000]. The temperature at different times is given by measurements at *n* points in the solid region, located at $r_k$, (k=1…n). On calling $T_k^m$, the measured temperatures, and $T_k^c$, the calculated temperature at those points, the solution of the present inverse problem can be obtained by minimizing the following fitness function

$$S = S(\tau_1,....,\tau_m) = \sum_{k=1}^{n}\left(T_k^m - T_k^c\right)^2 = \min \tag{4}$$

where *n* is the total number of measured temperatures, i.e., the number of points multiplied the number of measurements at each point. The inverse problem is recast as an optimization problem. A variety of numerical and analytical techniques have been developed to solve the optimization problems.

## 4 The particle swarm optimization algorithm

The Particle Swarm Optimization (PSO) algorithm introduced by Kennedy and Eberhart [Kennedy 1995, Eberhart 1998] in 1995 is a stochastic optimization technique which draws inspiration from the social behavior of a flock of birds or the collective intelligence of a group of social insects with limited individual capabilities. The basic PSO model consists of a swarm of M particles moving in a problem search space. Each particle is a potential solution of the global optimum over a given domain D. For a N-dimensional search space, the position of the i$^{th}$ particle is represented as $X_i = (x_{i1},x_{i2}, . . . ,x_{iN})$. At each generation, the new particle position is found by adding a displacement to the current position where the displacement is the particle velocity multiplied by a time step of one as shown in Eq. (5)

$$X_i^{n+1} = X_i^n + V_i^{n+1} \tag{5}$$

In Eq. (5), $X_i^n$ and $X_i^{n+1}$ represent the current and previous positions of particle i, $V_i^{n+1}$ i is the current velocity of particle i and is represented as $V_i^{n+1}= (v_{i1},v_{i2}, . . . ,v_{iN})$. The velocity of each particle is also updated at each generation and is given by:

$$V_i^{n+1} = V_i^n + c_1 r_1(P_{best,i} - X_i^n) + c_2 r_2(G_{best} - X_i^n) \tag{6}$$

where $V_i^n$ and $V_i^{n+1}$ are the current and previous velocities of the particle i, respectively. Each particle maintains a memory of its previous best position, say $P_{best,i} = (p_{i1}, p_{i2}, . . . ,p_{iN})$, where the position giving the best fitness function value. The best one among all the particles in the swarm is represented as the global best position, say $G_{best} = (p_{g1},p_{g2}, . . . ,p_{gN})$. The new velocity in Eq. (6) can be seen as the sum of three parts. The first part of Eq. (6) represents the previous velocity and is called the momentum part. The second part of the Eq. (6) represents the tracking of best position for individual particle and is called the cognition part. The third part of the Eq. (6) represents the cooperation among particles in the swarm and is called the social component. The cognitive learning coefficient $c_1$ and social learning coefficient $c_2$ are known as accelerating factors, and $r_1$ and $r_2$ are two random numbers generated by the uniform distribution within 0 and 1. The relative sizes of these components determine their contribution to the new particle velocity. The most common setting for $c_1$ and $c_2$ are 2.0 for the standard PSO algorithm.

One difficulty encountered in the standard PSO algorithm is that it could easily fall into local optimum in many optimization problems. Better selection of the inertial weight provides a balance between global and local exploration and exploitation. Therefore, less iteration is needed to find the optimal solution and improve the performance of the algorithm. A constriction factor was incorporated into the PSO algorithm by Clerc's suggestion [Clerc 1999] to insure the convergence of the algorithm (PSOC). The velocity term was update as follows:

$$V_i^{n+1} = C_3(V_i^n + c_1 r_1(P_{best,i} - X_i^n) + c_2 r_2(G_{best} - X_i^n)) \qquad (7)$$

When Clerc's constriction method is used, the constant multiplier $C_3$ is approximately equal to 0.7298 and the two coefficients $c_1$ and $c_2$ are 2.05. The Clerc's approach has been used for the determination of Heat Transfer Coefficients.

## 5 Computation procedure for the Particle Swarm Optimization

The aim of the inverse analysis is to iteratively estimate the unknown Heat Transfer Coefficients using the PSO procedure which results a negligible difference between measurements taken at the given locations of the work piece and temperatures computed from the numerical model. The fitness functions value of each particle at the nth iteration is given by the difference between the measured and calculated temperature curves, Eq (4) at the position $X_i^n$. The computational steps of the PSOM algorithm described above are given as follows:

- Step 1: Generate the initial particles in a swarm by randomly generating the position and velocity for each particle.
- Step 2: Evaluate the fitness function of each particle.
- Step 3: Update the Pbest,i for each particle, if its fitness is smaller than the fitness of its previous best position (Pbest,i).
- Step 4: Update the Gbest, if the fitness function of a particle is smaller than the fitness of the best position of all particles (Gbest).
- Step 5: Update each particle according to Eqs. (5) and (7).
- Step 6: Repeat the loop until the stopping criteria or a predefined number of generations is reached.

It is strongly advised to parallelize the computational jobs in Step 2 due to the fact that there are no interferences between the iterations as well as there is no communication between the particles in a given iteration. Therefore, these parts are executable in a data parallel fashion, which is ideal for GPU implementation [Szénási 2015, Kirk 2010]. We used the following configurations for the tests:

**GPU configuration**

- Graphics accelerator: NVIDIA Tesla K40c
- Architecture: Kepler (GK110B)
- Number of shades: 2880
- SMX Count: 15
- Memory: 12GB GDDR5

## 6 Numerical examples and discussion

In order to test the performance of the PSO algorithms on the estimation of the Heat Transfer Coefficients, numerical experiments have been performed. In the analysis, there was no physical set-up to directly measure the temperature $T_k^m$. Instead, theoretical Heat Transfer Coefficient functions as function of local coordinate and time have been defined, HTC(z,t) and substitute them directly into the equations (1)–(3) to calculate the temperatures at each location for the thermocouples (TC). The results are used in the computed temperature $T_k^m$ curves. Due to this concept the $T_k^m$ curves have been assumed to be error-free samples. The following concepts have been used for the computational investigations:

- The theoretical HTC(z,t) functions have been determined
- The Tkm temperature signals have been generated by obtaining simulations on the basis of HTC(z,t) functions
- Inverse computations have been carried out by applying PSO algorithm, in order to reconstruct the original HTC(z,t) functions
- The computational results were evaluated

The quenching process for a cylindrical work piece, mounted with 5 TC's was investigated. A 2D axis-symmetric heat transfer model was applied to calculate the temperature distribution during the cooling process. The physical properties of Inconel 600 alloy were assigned to the virtual work piece (Table 1). The thermocouples were assumed to be mounted at 1 mm below the side surface of the rod. The location of the TC's (the distances from the bottom of the cylinder) and the parameters used for the calculations are summarized in Table 2.

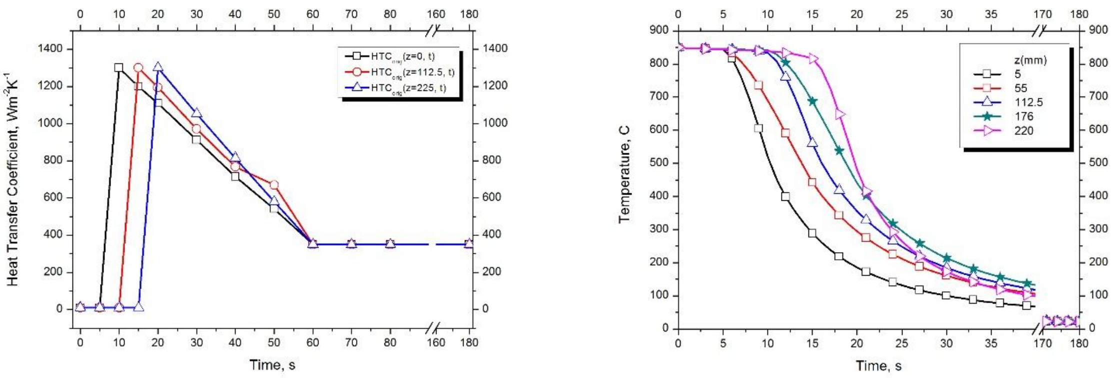
| **Temperature, T (C)** | **Heat Conductivity, k (W/mK)** | **Specific Heat, Cp (kJ/kgK)** | **Density, ρ (kg/m3)** |
|---|---|---|---|
| 27 | 14,8 | 0,444 | 8420 |
| 95,45 | 15,8374 | 0,4801 | 8420 |
| 195,95 | 17,3606 | 0,5038 | 8420 |
| 205,15 | 17,5 | 0,5038 | 8420 |
| 346,75 | 19,7721 | 0,5041 | 8420 |
| 554,15 | 23,1 | 0,5453 | 8420 |
| 596,15 | 23,8 | 0,5536 | 8420 |
| 662,15 | 24,9 | 0,5958 | 8420 |
| 796,45 | 27,1383 | 0,6817 | 8420 |

Table 1: The material properties of ISO 9950 alloy

The effect of wetting front kinematics that occurs during immersion quenching, is taken into consideration, by defining the heat transfer coefficient [Majorek 1994, Tensi 1992] HTC(z,t). The Heat Transfer coefficient function is assumed to be dependent on time and the vertical local coordinate (z). The theoretical HTC(z,t) as predefined, is represented at Fig. 1a., while the cooling curves obtained at the TC locations are shown at Fig. 1b. These curves have been applied as $T_m(t)$ temperature samples to calculate the fitness values for each particle in the swarm at each iteration. The HTC(z,t) is used for all the surfaces of the work piece, including the top and the bottom faces.

| Parameter | Value |
|---|---|
| Radius, R | 10 mm |
| Length, L | 225 mm |
| Initial temperature, $T_0$ | 850 C° |
| Ambient temperature, $T_{am}$ | 20 C° |
| Locations of TC 1-5 below the surface | r=R-1 |
| TC 1 | z = 5 mm |
| TC 2 | z = 55 mm |
| TC 3 | z = 112.5 mm |
| TC 4 | z = 167 mm |
| TC 5 | z = 220 mm |

Table 2: Parameters applied for the computational example




a b

Figure 1: The theoretical HTC(z,t) applied (a) and the cooling curves (b) obtained at given distance measured from the bottom of the probe

The swarm size used to reconstruct the Heat Transfer Coefficient function was chosen to M=1000 and the number of generations computed was defined in 500. We defined 45 parameters of the HTC(z,t) (15 time instances in 3 distances from the bottom of the probe) to be estimated, the particles were moving in a D=45 dimensional search space.

The predicted Heat Transfer Coefficient function obtained by the PSO technique is shown in Fig. 2. The calculated values of HTC(z,t) show a good agreement with the original (predefined) function at the time domain 0-40 s. The fluctuation of the results can be observed in the time period 40-180 s (when the original HTC function was determined on constant value). The temperature signals, $T_m(t)$ were calculated by using the $HTC_{orig}(z,t)$ and the estimated cooling curves $T_c(t)$ were obtained by $HTC_{PSO}(z,t)$ at the positions regarding Table 2. are presented in Fig 3. Satisfactory agreement of original and predicted cooling curves can be observed. The difference between the measured and estimated samples as a function of time for each TC positions is shown in Fig. 4. In order to quantify the magnitude of deviation between the preliminary defined and the recovered signals the mean and maximum value of the difference of cooling curves in each positions were calculated (Fig. 4).

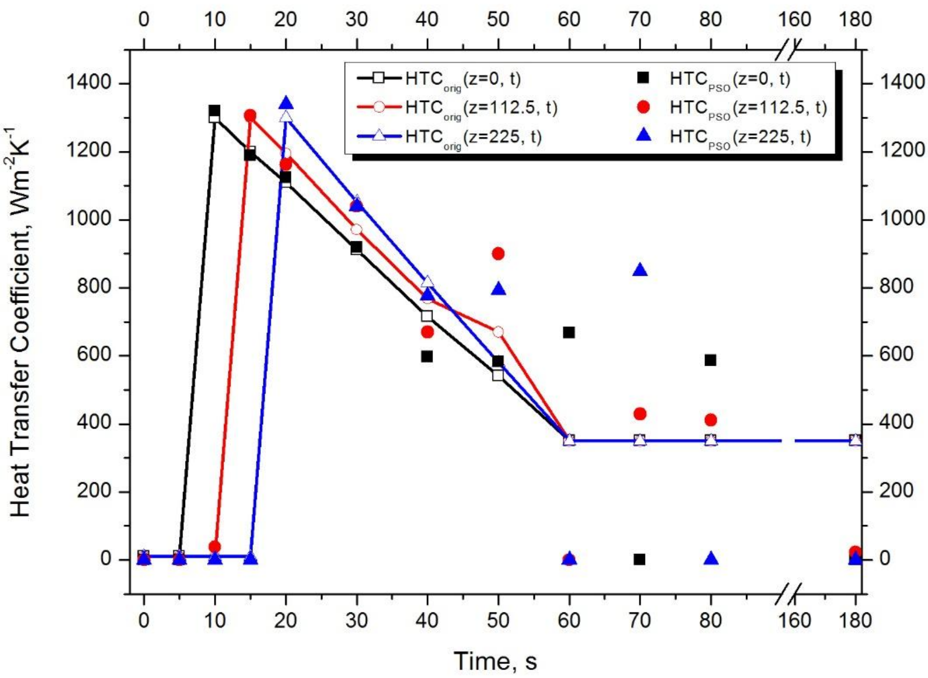



Figure 2: The original (HTC$_{orig}$) and estimated (HTC$_{PSO}$) Heat Transfer Coefficient functions

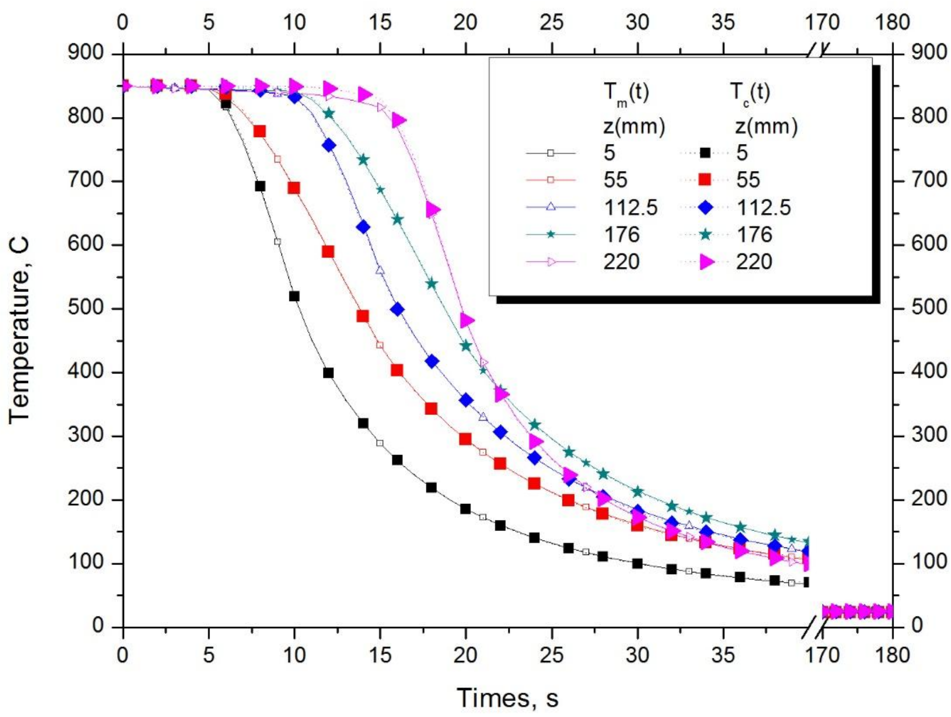



Figure 3: The original, $T_m(t)$ and estimated $T_c(t)$ cooling curves at given positions (z) of the probe

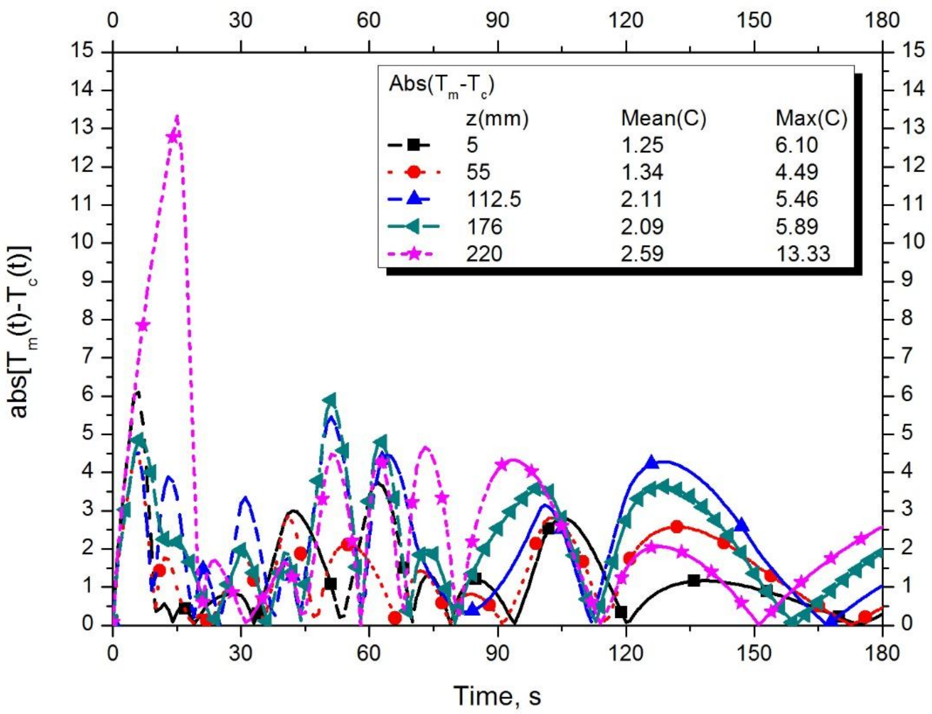



Figure 4: Difference between measured and calculated cooling curves at given positions (z) as a function of time as well as mean and maximum of the cooling curves

The highest value of temperature difference (13.33 C) was given close to the top surface of the cylinder (z = 220 mm) at 17.03 s. The maximum of differences at the further locations were approximately 6 C. The mean value of the differences were given between 1.25 and 2.59. Due to the low value of differences the PSO method applied to estimate the complex Heat Transfer Coefficient in a two dimensional axis-symmetrical model seems to be a feasible approach providing the proper accuracy.

It is important to report the results of the computational time required for PSO based estimation. The calculations have been made in:

1. Sequential processing (the fitness value of each particle has been estimated one after another in the swarm in one generation)
2. Multi-tier processing (8 parallel computation of particles at the same time)
3. Full parallelization on GPU architecture

The implementation of version 1 and 2 a PC hardware setup (Processor: Intelr CoreTM i7-2600, Architecture: Sandy Bridge, Number of cores: 4, Memory: 16GB DDR2) has been applied while for version 3 the GPU setup is used mentioned earlier. The time measured during performing the inverse estimations are summarized in Table 3. The parameters show the solution performed on the GPU architecture provides a significant acceleration of calculation time.

| PSO Implementation | Calculation time |
|---|---|
| Sequential processing | 22.47 |
| Multi-tier processing | 6.08 |
| GPU processing | 0.56 |

Table 3: The calculation times (in hours) required for HTC estimations

# 7 Conclusions

An inverse analysis using a Particle Swarm Optimization algorithm suggested by Clerc has been presented to identify Heat Transfer Coefficient in a two dimensional heat conduction problem. The Heat Transfer Coefficient obtained on the surfaces of a cylindrical probe was considered as functions of local coordinates and time. The obtained results underline the feasibility of the procedure and the capabilities for the PSO technique to reconstruct a complex surface Heat Transfer Coefficient without using any prior information of the unknown transient functions. The PSO algorithm has been carried out in high performance GPU configuration using the CUDA environment. The GPU implementation of the inverse heat conduction problem provides significant acceleration of the prediction compared to sequential or multi-tier computation used on a personal computer.

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