Fuzzy systems for engineers

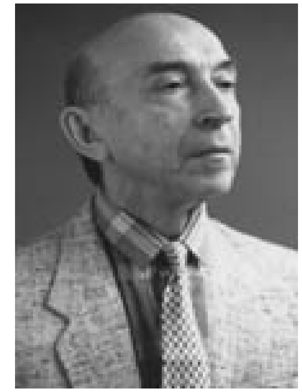
Introduction

The subject proposals

- 3 ECT (European Credit Transfer)
  - (or +2)
- Individual projects
- Script
- Exam

In F08

or 2.12 laboratory



By Zadeh, (Lotfi Zadeh, professor at the University of California at Berkley, honorary prof. at the Budapest Tech)

- artificial intelligence
- computational intelligence
- Soft computing (neural networks, genetic algorithms, fuzzy systems)

- The basic ideas underlying soft computing in its current incarnation have links to many earlier influences, among them Prof. Zadeh’s:
- 1965 paper on fuzzy sets;
- the 1973 paper on the analysis of complex systems and decision processes; and
- the 1979 report (1981 paper) on possibility theory and soft data analysis.

WHERE DID FUZZY LOGIC COME FROM?

The concept of Fuzzy Logic (FL) was conceived by Zadeh (1965), and presented not as a control methodology, but as a way of processing data by allowing partial set membership rather than crisp set membership or non-membership.

This approach to set theory was not applied to control systems until the 70's due to insufficient small-computer capability prior to that time.

Professor Zadeh reasoned that people do not require precise, numerical information input, and yet they are capable of highly adaptive control.

If feedback controllers could be programmed *to accept noisy, imprecise input, they would be much more effective and perhaps easier to implement.*

Unfortunately, U.S. manufacturers have not been so quick to embrace this technology while the Europeans and Japanese have been aggressively building real products around it.

FL provides a simple way to arrive at a definite conclusion based upon vague, ambiguous, imprecise, noisy, or missing input information.

FL's approach to control problems mimics how a person would make decisions, only much faster.

We are satisfied by “enough good” results.

WHAT are FUZZY LOGIC applications?

- FL is a problem-solving control system methodology that lends itself to implementation in systems ranging from simple, small, embedded micro-controllers to large, networked, multi-channel PC or workstation-based data acquisition and control systems.
- It can be implemented in hardware, software, or a combination of both.

Engineering problems

- Without analytical or numerical algorithms
- large computer capacity with (or without) intelligence
- “Infinite” computation time

- Image processing
- Large, multi-parametrical control problems
- ...

Medical diagnostic,
Complex game constructions,
Browser problems,...

- A real world is full with complex systems, with more information, than man can accept in a moment
- Human thinking is based on *approximate reasoning, like the*
- *FUZZY approximate reasoning,* based on *fuzzy logic.*

Uncertainty - fuzzy

- bizonytalan

- elmosódott,
- homályos,
- ingadozó,
- változó,
- határozatlan,
- kockázatos,
- pontatlan,
- változékony,
- véletlen,
- nehezen meghatározható,
- nem pontos,
- nem szabatos

Probability theory
valószínűség

Possibility theory

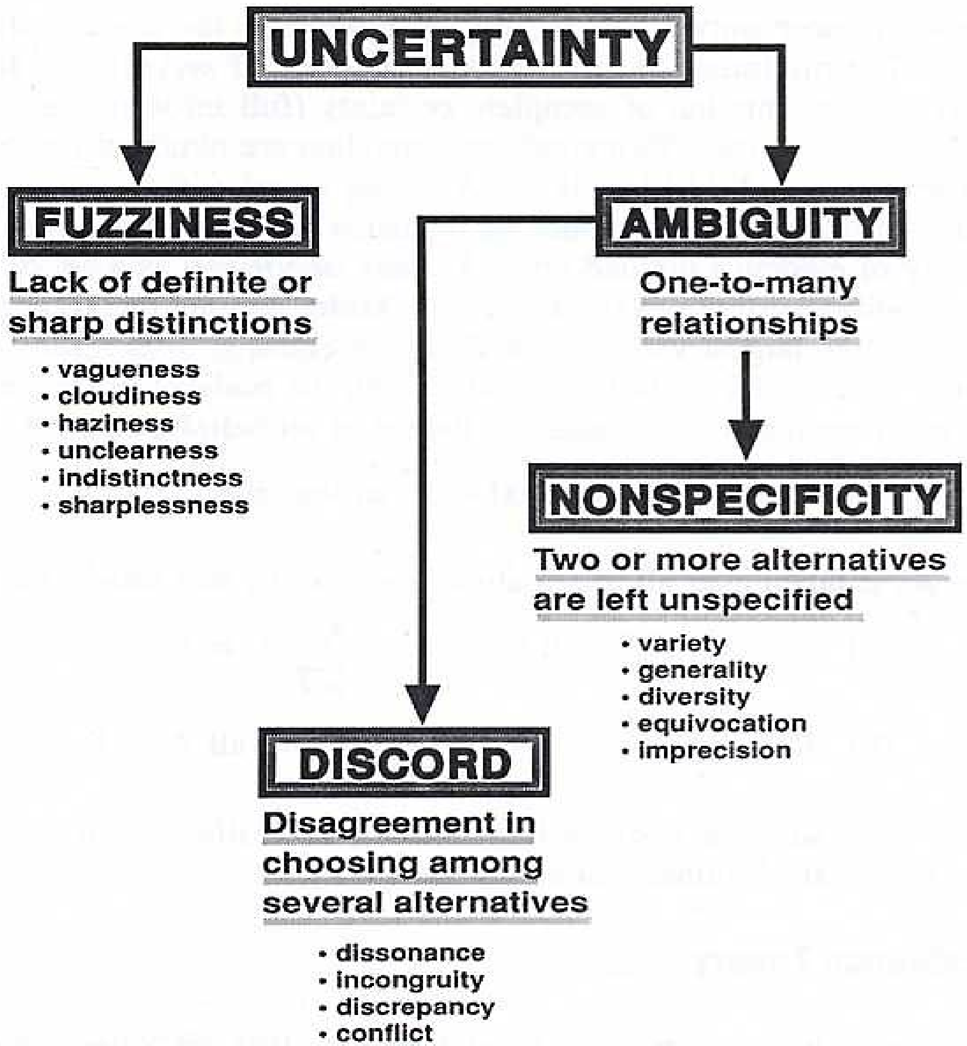
Uncertainty handling by mathematical tools

- fuzzy logic fuzzy sets
- kétértelműség - ambiguity,
- Pontatlanság – imprecision
- Missing information

Fuzzy

- The scientific means:
- Object,
  - Without crisp bounds
  - With uncertainity
  - not precise
  - Not exactly defined meaning

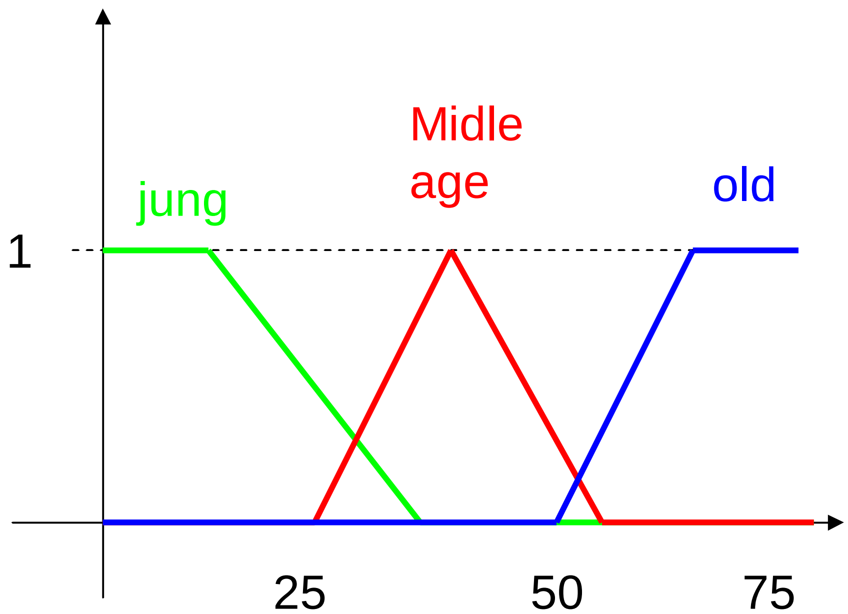
(figure from Klir&Yuan)



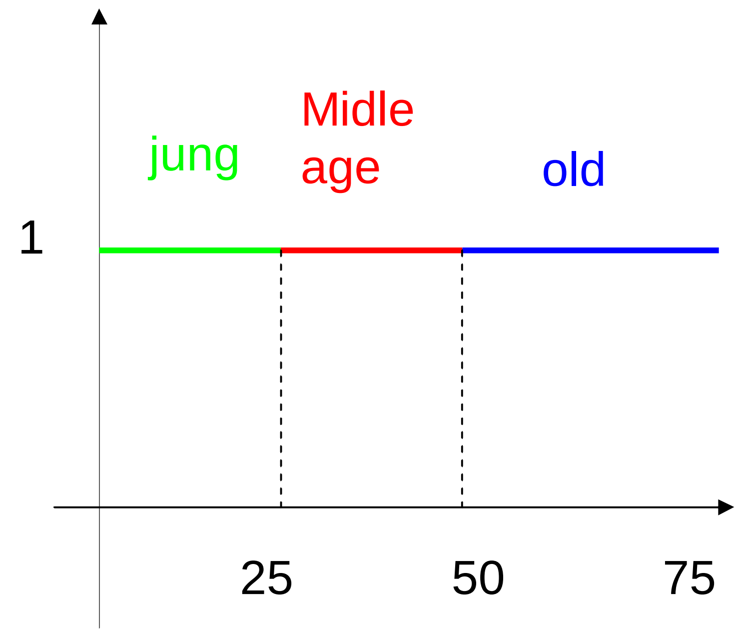
Three basic types of uncertainty.

Example: human Ages

With membership functions



Classical (characteristic) form



Sets,
Operation on sets

Sets and Sets and Numbers

Definition: A set is any collection of objects.

Set Notations

- Sets are commonly named by capital letters.
- Roster Form – simply list the elements in the set between curly brackets, {…}

Examples

- A = { 1, 2, 3, 7, e }
- The ellipsis (…) can be used to list an infinite set where the elements follow a pattern, i.e. B = {0, 2, 4, 6, 8, 10, …)
- Interval Notation – when indicating a set that contains and uncountable number of elements, we can describe the set of numbers by giving the endpoints of the interval.

Examples

- (0, 1) - every number between 0 and 1, but NOT INCLUDING 0 and 1
- [0, 1] – every number between 0 and 1, and INCLUDING 0 and 1
- Intervals may be of the form (a, b] or [a, b) as well; The union symbol may also be used to indicate more than one interval in a set.
- $D = (0,1) \cup (4,7]$ - means D is the set of all number either between 0 and 1 (not including 0 and 1) OR the number between 4 and 7 (not including 4).

Set Builder Notation

Describes the set using a variable and stating the mathematical

- properties of the elements in the set,

A = {x: "properties of the elements represented by x"}

- Examples

P = { z : z > 0 } means all positive numbers

E = { x : x ≥ 0 and x is an even integer} means the same thing as set B above under roster form, B = {0, 2, 4, 6, 8, 10, ...).

Set Symbols

- $\in$ - “is an element of” or “is in the set”
  If A = {1, 2, 3}, then 1 $\in$ A.
- $\notin$ - “is NOT an element of” or “is NOT in the set”
  If A = {1, 2, 3}, then 17 $\notin$ A.
- $\cup$ - This is the symbol for Set Union. Union means “or” and is like addition.
  If A = {1, 2, 3} and B = {3, 4, 5}, then A “or” B = A $\cup$ B = {1, 2, 3, 4, 5}
- $\cap$- This is the symbol for Set Intersection. Intersection means “and”, and tells us to find the elements in common between two sets.
  If A = {1, 2, 3} and B = {3, 4, 5}, then A$\cap$B = {3} since 3 is the only element that sets A and B have in common.
- \ - The is the symbol for set subtraction.
  If A = {1, 2, 3} and C = {3}, then A \ B = {1, 2}.
- Notation of the empty set is $\varnothing$.

The Hierarchy of Sets of Number

- N = {1,2,3,...}

This set is the set of Natural Numbers or Counting Numbers.

- W = {0,1,2,3,...}

This set is the set of Whole Numbers. Notice that $W = N \cup \{0\}$

- $Z = \{\ldots,-4,-3,-2,-1,0,1,2,3,4,\ldots\}$

This set is the set of Integers. Note that we can also use superscripts to indicate just the positive integers or the negative integers, Z+ and Z-. Note also that Z+ = N.

$$Q = \left\{ \frac{p}{q} \middle| p \in Z, q \in Z, q \neq 0 \right\}$$

This set is the set of Rational Numbers, or more informally the set of fractions. Notice that the notation for Q uses set-builder notation.

The definition says that rational numbers are fractions where the numerator can be any integer and the denominator can be any integer except 0

- R all real numbers

The set of Real Numbers include all of the above sets, and also the irrational numbers. Irrational Numbers are the numbers which are not rational, i.e. which cannot be written as a fraction.

Some examples of irrational numbers are: π, e, and $\sqrt{2}$.

- I = irrational numbers Note:

R = Q È I : The real numbers equal the rational numbers union ("plus") the irrational numbers

I = R \ Q : Irrational numbers equal the reals minus rationals

Q = R \ I : Rational numbers equal the reals minus irrationals

- Complex numbers

Classical sets

- *X* is a crisp set if it is clear (wthout uncertanity) is an element in the set or not:

25 $\in$ [25; 40], de 24,9999999 $\notin$ [25; 40*A*].

- *Notations*

$x \in X$, $x \notin X$, $A \subseteq X$, $A = B$, $\emptyset$

- Operaitons on sets

  $A \cap B$, $A \setminus B$, $A \cup B$, $\bar{A}$

- Properties of the operations
  - Kommutativity ($A \cap B = B \cap A$);
  - ($A \cap (B \cap C) = (A \cap B) \cap C$);
  - ($A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$);
  - idempotenci ($A \cup A = A$);
  - The neutral element ($A \cup \emptyset = A$, $A \cap X = A$);
  - ($A \cup A = X$);
  - ($A \cap \bar{A} = \emptyset$);
  - De Morgan rules

Characteristic function

$$c_A(x) = \begin{cases} 1 & ha \quad x \in A \\ 0 & ha \quad x \notin A \end{cases}$$

Min, max for char. function

$$c_{A \cap B}(x) = \min(c_A(x), c_B(x))$$

$$c_{A \cup B}(x) = \max(c_A(x), c_B(x))$$

$$c_{\overline{A}}(x) = 1 - c_A(x)$$

$A \subseteq B$ if and only if $A(x) \leq B(x)$, $\forall\, x \in X$

Other operations

$$c_{A\cap B}(x) = c_A(x) \cdot c_B(x)$$

$$c_{A\cup B}(x) = c_A(x) + c_B(x) - c_A(x) \cdot c_B(x)$$

$$c_{\overline{A}}(x) = \sqrt{1 - (c_A(x))^2}$$

$$c_{A\cap B}(x) = \max(c_A(x) + c_B(x) - 1, 0)$$

$$c_{A\cup B}(x) = \min(c_A(x) + c_B(x), 1)$$

$$c_{\overline{A}}(x) = \left(1 - \sqrt{c_A(x)}\right)^2$$

Membership functions

- The membership function is a graphical representation of the magnitude of participation of each input.

For $X \neq \emptyset$

$$m_A(x): X \rightarrow [0,1]$$

Is the membership function

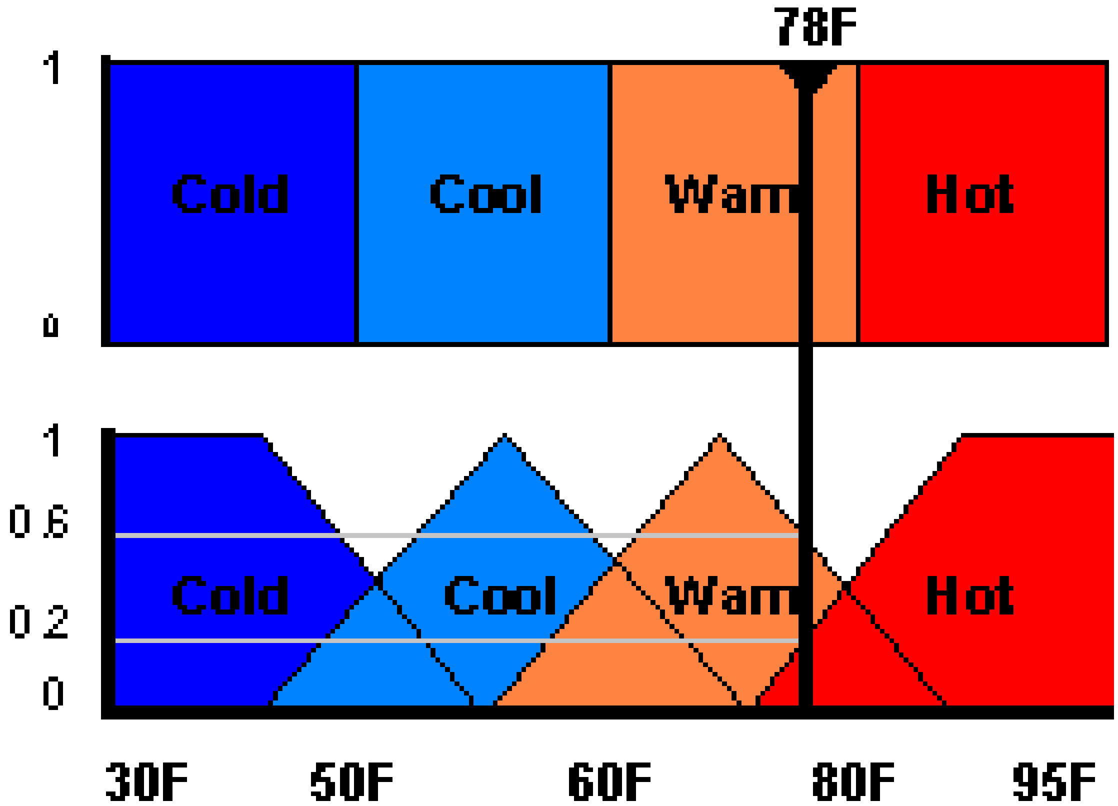
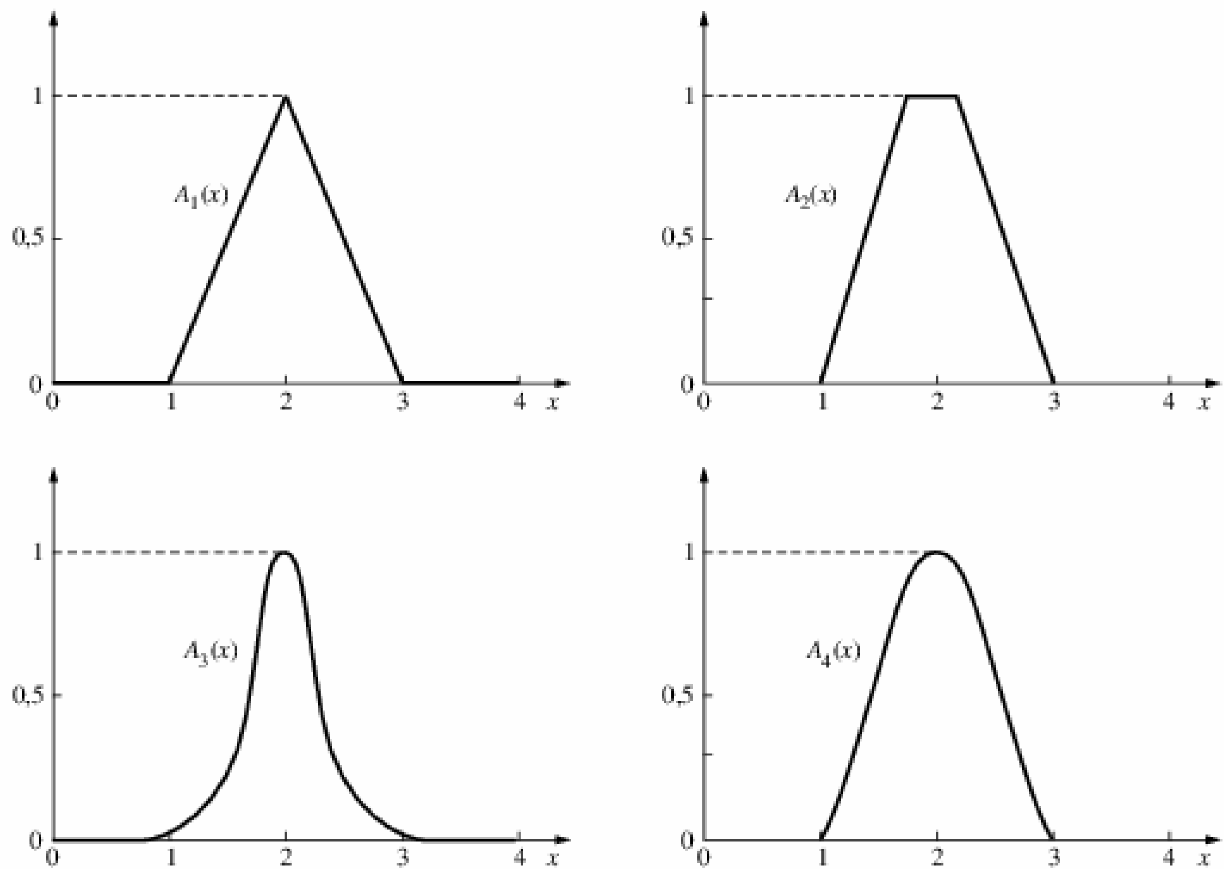


Figure2: Conventionnal and Fuzzy Sets

- From http://www.aptronix.com/fide/howfuzzy.htm

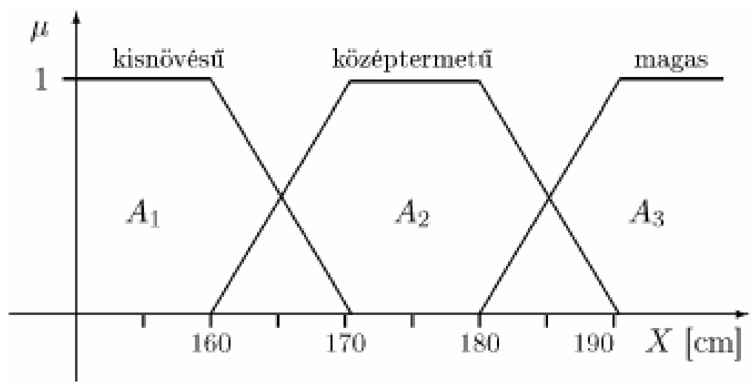
About 2



Fuzzy sets
Basic notations

Membership functions

$$A_1(x) = \begin{cases} 1 & ha \quad x \le 160 \\ \dfrac{(170 - x)}{10} & ha \quad 160 < x < 170 \\ 0 & ha \quad x \ge 170 \end{cases}$$

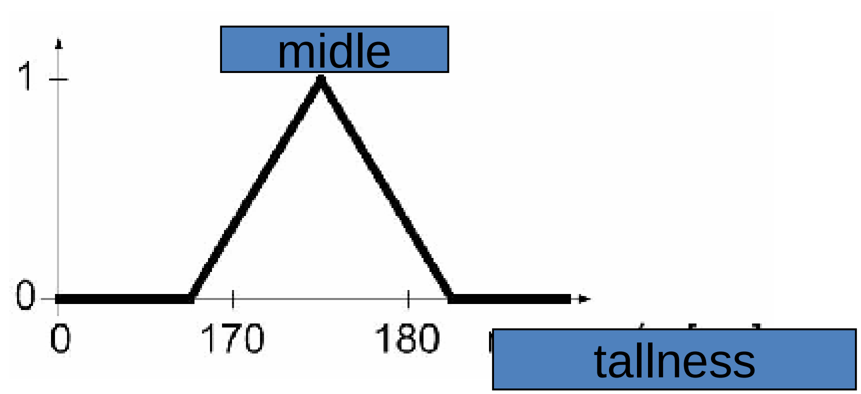


$$A_3(x)=\begin{cases} 0 & ha \quad x \le 180 \\ \dfrac{(x-180)}{10} & ha \quad 180 < x < 190 \\ 1 & ha \quad x \ge 190 \end{cases}$$

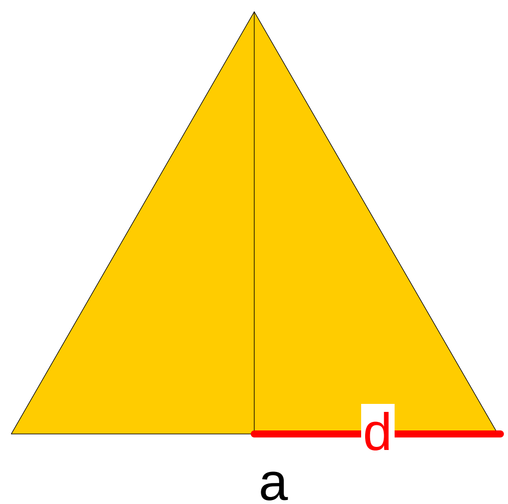
$$A_2(x)=\begin{cases} 0 & ha \quad x \le 160 \; vagy \; x \ge 190 \\ \dfrac{(x-160)}{10} & ha \quad 160 < x < 170 \\ 1 & ha \quad 170 \le x \le 180 \\ \dfrac{(190-x)}{10} & ha \quad 180 < x < 190 \end{cases}$$

Triangular fuzzy sets (tallness)

$$midle(x) = \begin{cases} 1 - \dfrac{|x - 175|}{10} & if \quad 165 \le x \le 185 \\ 0 & otherwise \end{cases}$$



$$A(x) = \begin{cases} \max\left(1 - \dfrac{|x-a|}{d}, 0\right) & \text{if } d \neq 0 \\ c_a(x), & \text{if } d = 0 \end{cases}$$



Support of the fuzzy set

$supp(A) = \{ x \in X \mid A(x) > 0 \}$

$supp(A_1) =]m, 170[$

$supp(A_2) =]160, 190[$

$supp(A_3) =]180, M[$

Core of the Fuzzy set

$core(A) = \{ x \in X \mid A(x)=1 \}$

$core(A_1) = [\mathrm{m}, 160]$
$core(A_2) = [170, 180]$
$core(A_3) = [190, \mathrm{M}]$

α-level cut

Let *A* be a fuzzy set on the universe *X* and $\alpha \in [0, 1]$. The α-level cut of the fuzzy set *A* is the

$$[A]^a = \begin{cases} \{t \in X \mid A(t) \geq a\} & if \quad a > 0 \\ cl(suppA) & if \quad a = 0 \end{cases}$$

Set, where *cl*(*supp A*) is the closed interval of the support of *A*.

$[A1]^{\alpha} = [m; 170-10\alpha]$
$[A2]^{\alpha} = [160 + 10\alpha; 190-10\alpha]$
$[A3]^{\alpha} = [180 + 10\alpha; M]$

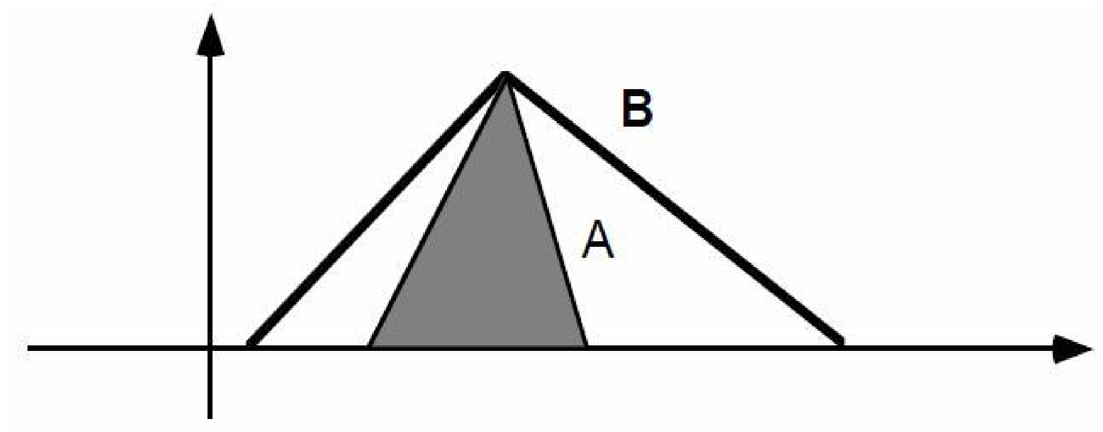
Hight of the Fuzzy set

$$h(A) = \sup_{x \in X} A(x)$$

- The fuzzy set is normal, if $h(A) = 1$. Otherwise it is subnormal.

Fuzzy subset

$A \subseteq B$, if $A(t) \leq B(t)$ for all $t \in X$.



Equality

$A = B$, if $A(t) = B(t)$ for all $t \in X$.

- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
- $\emptyset \subseteq$ A.
- $A \subseteq X$.
- $\emptyset(x) = 0$ for all $x \in X$.

MATLAB exercise

- Plot different membership functions in the MATLAB environment.

Further sources

- http://www.seattlerobotics.org/Encoder/mar98/fuz/flindex.html